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MOTION OF MISCIBLE AND IMMISCIBLE FLUIDS IN CLOSED HORIZONTAL AND VERTICAL DUCTS

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Abstract—A theory of large lossless waves with two fluids in horizontal closed channels of arbitrary cross-section is developed. The dynamic conditions for infinitesimally small disturbances is derived from this theory and it is shown that the dynamic condition for waves of finite magnitude is in agreement with Long's (1956) first order estimate for small waves in channels of rectangular cross-section. It therefore appears probable that an adequately accurate dynamic condition is available for such waves of all sizes.

Results from the theory are used to quantitatively explain experimental results by Wallis & Dobson (1973) for the onset of slugging in horizontal channels and by Leach & Thompson (1975) for the counter-current discharge of fluids along a horizontal pipes between closed tanks. In both cases an influence of the ratio of the densities of the phases upon the usually accepted Froude number, which already contains density correction factors, is found.

Transfer of miscible unstably stratified fluids through each other in vertical ducts is found to be described by a turbulent diffusion process which is controlled by the rate of energy dissipation and the duct diameter. The process is therefore quite different to that in horizontal ducts.

1. INTRODUCTION

Besides the presentation of various theoretical treatments, this paper is written to illustrate and stress the difference between closed horizontal and closed vertical channels with respect to the relative motion ot two fluids, where "closed" refers to the containment of a second phase rather than closure of the ends of the channel. The difference became apparent to the author during experiments carried out by J. Kubie at the Central Electricity Research Laboratories. Figure 1 shows water advancing into brine and brine advancing into water in a 32 mm bore horizontal tube closed at both ends. The two fluids were initially separated by a partition which was withdrawn from between the spring-loaded flanges half way along the tube. It is seen that the two fluids are unmixed and advance in the manner characteristic of long bubbles of immiscible fluids, as investigated experimentally by Zukoski (1966) and Gardner & Crow (1970). In the final state, Kubie's experimental tube contained two stationary layers of essentially unmixed fluids.

When Kubie inclined the tube, he found that waves between the advancing fluid layers tended to promote mixing. Thus figure 2 shows that the velocity of the fluid fronts in a horizontal tube was approximately in agreement with Zukoski's (1966) results for immiscible fluids but fell below Zukoski's prediction as the inclination increased. Figure 2 shows no results for an inclination greater than 60° since mixing became too pronounced for an advancing front to be observed.

The conditions that obtained with a vertical tube are best illustrated, because the photographic reproduction is better, by results for the case of carbon tetrachloride initially lying on top of water, as shown in figure 3. A spherical cap of water, which advanced according to Zukoski's prediction, is seen but generally the system comprises drops of water and drops of carbon tetrachloride. Not obvious from figure 3 is that the drops were in violent motion and similar packets of the original fluids were observed in experiments with brine–water systems. In the last case, however, where there was no spherical cap, the progress of each fluid into the other was much slower than for the horizontal tube and the very vigorous motions were observed for a long time after the two fluids in a horizontal tube would have come to rest as stably stratified layers. Clearly different theoretical approaches have to be made in considering horizontal and vertical tubes. Also, although similar approaches can be used for miscible and immiscible fluids in a horizontal channel, different approaches are required in a vertical channel.

The next section will develop equations for waves without energy losses in closed channels,

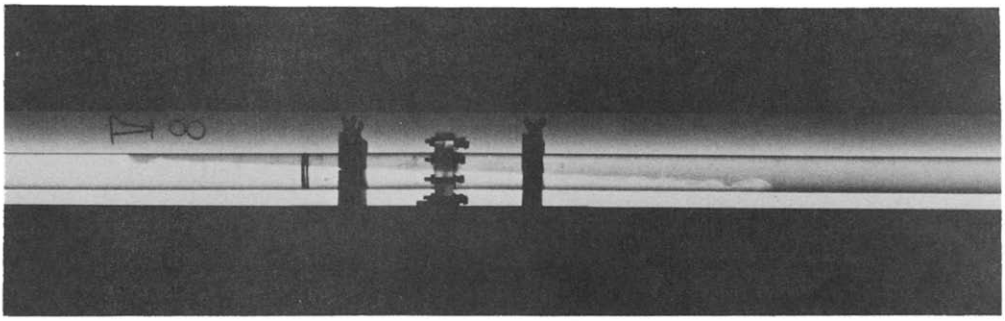

Figure 1. 5 Per cent brine and water moving through each other in 32 mm bore horizontal tube.

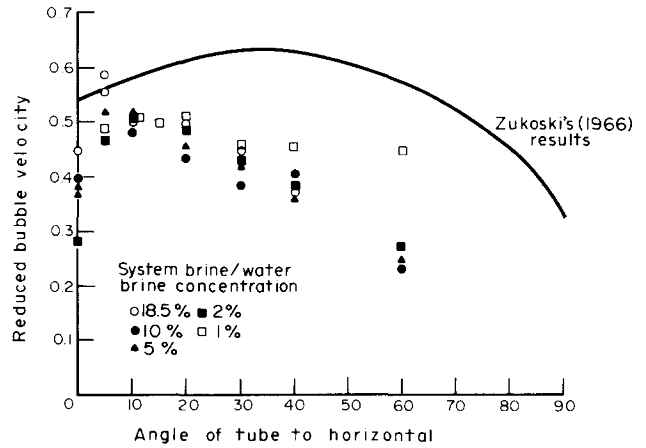

Figure 2.

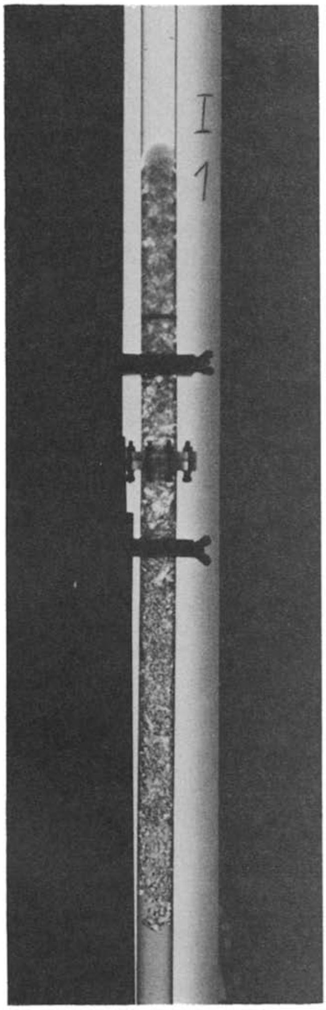

Figure 3.

Figure 2. Kubie's results for miscible liquids compared with Zukoski's correlation.

Figure 3. Water passing up through carbon tetrachloride in a vertical tube.

since it has been demonstrated by Yih (1965), Zukoski (1966) and Gardner & Crow (1971) that lossless systems describe the kinetics of flows in horizontal situations. One result will be an expression for the dynamics of an infinitesimally small wave, which will be used to explain some results of Mercer & Thompson (1975) for the countercurrent transfer of two miscible fluids between closed horizontal tanks, joined by a horizontal pipe. It will be seen that the flowrate depends not only on the modified Froude number

$$F_{QH} = \left[\frac{\rho_H}{\Delta\rho g d^5}\right]^{1/2} Q \qquad [1.1]$$

but also on the density ratio

$$P = \left(\frac{\rho_L}{\rho_H}\right) \qquad [1.2]$$

ρ_L and ρ_H are the densities of the light and heavy phases respectively, $\Delta\rho$ is the density difference between the phases, g is the acceleration due to gravity, d is the diameter of the pipe and Q is the volumetric flux of either phase.

The theory for the waves will also be used, with an additional hypothesis, to correlate the results of Wallis & Dobson (1973) for the onset of slugging in horizontal tubes. It will be seen that the correlation is rather better than that given by Wallis, which is presented later as [3.1], and, again, it implies a variation of the correlation with the density ratio. Therefore, extrapolation of Wallis' work at atmospheric pressure to, say, steam–water systems in high pressure boilers must be done with caution.

The explanation of the phenomena in vertical tubes with immiscible fluids assumes that the process is essentially one of turbulent diffusion. A hypothesis is presented for a relationship between the diffusivity and the energy dissipated by the movement of the fluids. Some experiments of Leach & Thompson (1975) are used for its confirmation.

2. LARGE WAVES IN CLOSED CHANNELS

Long (1956) theoretically examined small waves in closed rectangular channels and found them to be of sech^2 profile. Here we will specifically study large waves when the mathematical tools required are less sophisticated but are therefore applicable to channels of arbitrary cross-section. Moreover we will retrieve Long's most important results and thus indicate how they can be extrapolated through the whole size range.

The system is illustrated in figure 4. Stations 1 and 2 are chosen at a sufficient distance from the centre of the wave for velocities to be considered uniform. The depth of the lower heavy phase is h', its cross-sectional area is A' and it is designated, where necessary, by subscript H. The upper light phase is designated by subscript L, H is the total channel depth and A' is the total channel area. x_1' and x_2' are the distances of the centres of pressures of areas A_1' and A_2' from the two-phase interface and y_0' is the distance of the centre of pressure of area A_2' less area A_1' below the interface at station 2. p is the pressure at the interface.

Energy is conserved in each phase. Thus

$$p_1 + \frac{\rho_H v_{1H}^2}{2} + \rho_H g h_1' = p_2 + \frac{\rho_H v_{2H}^2}{2} + \rho_H g h_2', \qquad [2.1]$$

$$p_1 + \frac{\rho_L v_{1L}^2}{2} - \rho_L g(H - h_1') = p_2 + \frac{\rho_L v_{2L}^2}{2} - \rho_L g(H - h_2'). \qquad [2.2]$$

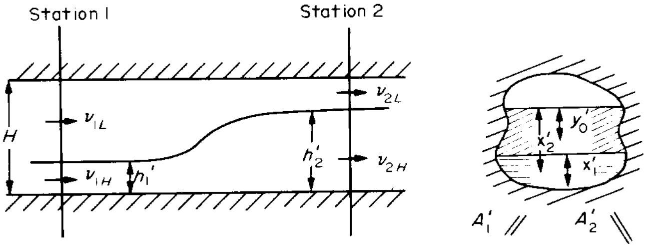

Figure 4. Definition sketch for large wave theory.

The dynamic force balance over the whole channel cross-section is

$$\rho_H v_{1H}^2 A_1' + \rho_L v_{1L}^2 (A' - A_1') + p_1 A' + \rho_H g A_1' x_{1H}' - \rho_L g (A' - A_1') x_{1L}'$$
$$= \rho_H v_{2H}^2 A_2' + \rho_2 v_{2L}^2 (A' - A_2') + p_2 A' + \rho_H g A_2' x_{2H}' - \rho_L g (A' - A_2') x_{2L}'. \quad [2.3]$$

Continuity relationships are

$$A_1' v_{1H} = A_2' v_{2H}, \quad [2.4]$$

$$(A' - A_1') v_{1L} = (A' - A_2') v_{2L}. \quad [2.5]$$

Also the following relationships concerning the centres of pressure can be derived by elementary means.

$$A_2' x_{2H}' - A_1' x_{1H}' = (A_2' - A_1') y_0' + (h_2' - h_1') A_1', \quad [2.6]$$

$$(A' - A_2') x_{2L}' - (A' - A_1') x_{1L}' = (A_2' - A_1') y_0' - (h_2' - h_1')(A' - A_1'). \quad [2.7]$$

We eliminate $(p_2 - p_1)$ between [2.1] and [2.3] and employ [2.4] and [2.7] to achieve

$$F_{1H}^2 \frac{A_1}{A_2} + \frac{F_{1L}^2}{(1 - A_2)^2} \left[\frac{A_1 + A_2}{2} - A_1 A_2 \right] - y_0 - A_1 \left[\frac{h_2 - h_1}{A_2 - A_1} \right] = 0 \quad [2.8]$$

and similarly we obtain from [2.2] and [2.3]

$$\frac{F_{1H}^2}{A_2^2} \left[\frac{A_1 + A_2}{2} - A_1 A_2 \right] + F_{1L}^2 \left[\frac{1 - A_1}{1 - A_2} \right] + y_0 - (1 - A_1) \left[\frac{h_2 - h_1}{A_2 - A_1} \right] = 0 \quad [2.9]$$

where

$$F_{1H}^2 = \frac{\rho_H v_{1H}^2}{\Delta \rho g H}, \qquad F_{1L}^2 = \frac{\rho_L v_{1L}^2}{\Delta \rho g H}, \quad [2.10]$$

$$h_1 = \frac{h_1'}{H}, \qquad h_2 = \frac{h_2'}{H}, \qquad y_0 = \frac{y_0'}{H}, \quad [2.11]$$

$$A_1 = \frac{A_1'}{A'}, \qquad A_2 = \frac{A_2'}{A'} \quad [2.12]$$

where $\Delta \rho = (\rho_H - \rho_L)$.

We finally derive from [2.8] and [2.9]

$$\frac{F_{1H}^2}{A_2^2} = \frac{2}{(A_2 - A_1)^2} [(1 - A_1)(h_2 - h_1) - (2 - A_1 - A_2) y_0], \quad [2.13]$$

$$\frac{F_{1L}^2}{(1 - A_2)^2} = \frac{2}{(A_2 - A_1)^2} [(A_1 + A_2) y_0 - A_1 (h_2 - h_1)]. \quad [2.14]$$

2.1 *Dynamic relationships for a small disturbance*

The following approximations can be written for the case of a very small disturbance

$$y_0 = \frac{h_2 - h_1}{2}, \quad [2.15]$$

when an area by itself is involved

$$A_1 = A_2, \quad [2.16]$$

when a difference in areas is involved

$$A_2 - A_1 = \frac{H w'}{A'} (h_2 - h_1), \quad [2.17]$$

where w' is the width of the interface. Thus [2.13] and [2.14] become

$$\frac{F_{1H}^2}{A_1^2}=\frac{F_{1L}^2}{(1-A_1)^2}=\left(\frac{A'}{Hw'}\right) \quad [2.18]$$

from which

$$|F_{1H}|+|F_{1L}|=\left[\frac{A'}{Hw'}\right]^{1/2}. \quad [2.19]$$

Equations [2.18] and [2.19] are dynamic relationships but they can be reduced to a more recognisable form by re-employing the equations of [2.18] in [2.17] to yield

$$\frac{\rho_H v_{1H}^2 w'}{\Delta\rho g A_1'}+\frac{\rho_L v_{1L}^2 w'}{\Delta\rho g(A'-A_1')}=1 \quad [2.20]$$

where it is noted that the subscript defining station 1 is only retained to differentiate between A' and A_1'.

Equation [2.20] for $A'=\infty$ reduces to the accepted form of

$$\frac{\rho_H v_{1H}^2 w'}{\Delta\rho g A_1'}=1 \quad [2.21]$$

for open channel flow.

For a rectangular channel the dynamic condition of [2.20] reduces to

$$\frac{\rho_H v_{1H}^2}{\Delta\rho g h_1'}+\frac{\rho_L v_{1L}^2}{\Delta\rho g(H-h_1')}=1 \quad [2.22]$$

which, rearranged, is the zero order approximation found by Long (1956) from the theory of small waves.

2.2 *Properties of large waves illustrated by reference to a rectangular channel*

Equations [2.13] and [2.14] reduce to very simple forms for rectangular channels.

$$\frac{F_{1H}^2}{h_2^2}=\frac{F_{1L}^2}{(1-h_2)^2}=1 \quad [2.23]$$

and also, of course,

$$\frac{F_{2H}^2}{h_1^2}=\frac{F_{2L}^2}{(1-h_1)^2}=1. \quad [2.24]$$

Thus the modulus of the modified Froude number for a phase at one station is equal to the reduced depth of that phase at the other station.

Other important properties of the wave can be derived in the following fashion. Define

$$F_\Delta=\left[\frac{\rho_L}{\Delta\rho g H}\right]^{1/2}(v_L-v_H). \quad [2.25]$$

From [2.23]

$$h_2=\left[\frac{\rho_H}{\Delta\rho g H}\right]^{1/2}|v_{1H}|. \quad [2.26]$$

We can derive directly from [2.23] or from [2.19]

$$|F_{1H}|+|F_{1L}|=1\,. \tag{2.27}$$

Let us assume the coordinate direction such as to make u_{1L} positive. Then [2.27] becomes

$$\left[\frac{\rho_L}{\Delta\rho g H}\right]^{1/2}(v_{1L}\mp|v_{1H}|)+\frac{\rho_H{}^{1/2}\pm\rho_L{}^{1/2}}{(\Delta\rho g H)^{1/2}}|v_{1H}|=1 \tag{2.28}$$

where the upper sign of a choice is accepted if v_{1H} is positive. Eliminate $|v_{1H}|$ between [2.26] and [2.28]

$$h_2=\frac{1-F_{1\Delta}}{1\pm\left(\dfrac{\rho_L}{\rho_H}\right)^{1/2}}\,. \tag{2.29}$$

Similarly

$$h_1=\frac{1-F_{2\Delta}}{1\pm\left(\dfrac{\rho_L}{\rho_H}\right)^{1/2}}\,. \tag{2.30}$$

Equations [2.29] and [2.30] can be used to draw the diagram of figure 5, in which the examples are for cases with v_{1H} being of opposite sign to v_{1L}, since this is the most common case. h at one station is a linear function of F_Δ at the other station, with $h=0$ corresponding to $F_\Delta=1$ and $h=1$ corresponding to $F_\Delta=\mp(\rho_L/\rho_L)^{1/2}$. Thus, with the signs of the two velocities being opposite, it is seen that the range of F_Δ, for which these waves can be discussed, becomes more restricted as the density of the light phase approaches that of the heavy phase.

It is easily seen from figure 5 how the height of the wave can be estimated if two of the parameters h_1, h_2, $F_{1\Delta}$ and $F_{2\Delta}$ are known. Smaller waves are possible but it is suggested that the largest possible lossless waves are described, since the wave of figure 4 can be joined far downstream by another wave which is the mirror image of the first. The whole wave thus formed, will have a flat top, which can be considered to be the result of increasing the height of Long's sech^2 profile waves until interference with the top of the channel is achieved.

A last point must be made that the physics of the waves which touch or almost touch the top of the channel, so that $h_2=1$ is doubtful. Reference to [2.23] and [2.24] shows that v_{1L}, as reason

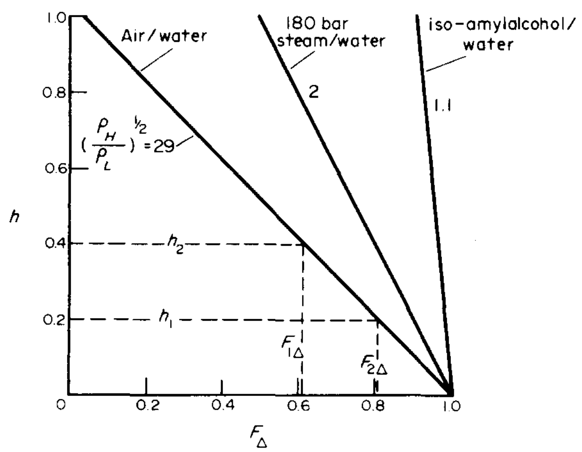

Figure 5. The $h-F_\Delta$ diagram.

dictates, becomes zero when $h_2 = 1$ but v_{2L} is non-zero. Mathematically this is consistent but physically it is implausible.

2.3 *Comparison of the results of the analysis with those of Long* (1956)

We take the result of [2.27] for a closed rectangular channel and substitute from [2.23] to obtain

$$\frac{F_{1H}^2}{h_2}+\frac{F_{1L}^2}{1-h_2}=1\,. \qquad [2.31]$$

This is the dynamic condition for a large wave and has been derived in the same way as [2.22] for a small disturbance. Equation [2.31] can be rearranged to yield

$$\frac{F_{1H}^2}{h_1}\left[1+\frac{h_2-h_1}{h_1}\right]^{-1}+\frac{F_{1L}^2}{(1-h_1)}\left[1-\frac{h_2-h_1}{1-h_1}\right]^{-1}=1\,. \qquad [2.32]$$

Equation [2.32] can be compared with the first order approximation of Long's (1956) analysis, the zero order approximation having already been given as [2.22]. The approximation is

$$\frac{F_{1H}^2}{h_1}\left[1-\frac{h_2-h_1}{h_1}\right]+\frac{F_{1L}^2}{(1-h_1)}\left[1+\frac{h_2-h_1}{1-h_1}\right]=1 \qquad [2.33]$$

which agrees with [2.32] within the accuracy of the approximations made. Long continued to evaluate a complicated second order approximation to his theory but, it is to be suspected that [2.31] is sufficiently accurate for most purposes over the whole range of wave heights.

3. ONSET OF SLUGGINGS

Wallis & Dobson (1973) reviewed existing experimental work and carried out further detailed experiments in a closed rectangular channel to determine the onset of slugging. This is defined as follows. In all experiments air was blown over water which was essentially stationary and, if the air velocity was increased, a critical, well-defined value was found at which the large waves lifted to the top of the channel and were blown along with the air. Wallis correlated the critical value by

$$F_{1\Delta c}=0.5(1-h_1)^{1/2} \qquad [3.1]$$

but he derived this correlation by reasoning which took no account of the flow of the heavy phase in the channel below the level h_1. Here a hypothesis will be presented which correlates the air–water results equally well or even better than [3.1] and which implies that [3.1] may fail when applied to systems in which the densities of the phases approach each other. The steam–water system at 180 bar is of particular interest to the author and then $\rho_L/\rho_H = 0.25$.

Consider a wave of the form shown in figure 4 but with co-ordinates chosen such that v_{1H} is stationary and the wave is therefore moving to the right. Estimate the total energy flux difference, Δe, between station 2 and station 1. The algebra involved is considerable and will not be given but the result is

$$\Delta E=F_{1\Delta}[1-F_{1\Delta}]\left[1-\left(1-\frac{\rho_L}{\rho_H}\right)^{1/2}h_1-F_{1\Delta}\right]\left[\left(1+\left(\frac{\rho_H}{\rho_L}\right)^{1/2}\right)F_{1\Delta}-2\right], \qquad [3.2]$$

$$\Delta E=\frac{2\Delta e(\rho_H{}^{1/2}-\rho_L{}^{1/2})^3}{H(\rho_L\rho_H)^{1/2}(\Delta\rho gH)^{3/4}}\,. \qquad [3.3]$$

Values of ΔE are plotted in figure 6 for $(\rho_H/\rho_L)^{1/2}=29$. Each curve is for a fixed value of h_1 but

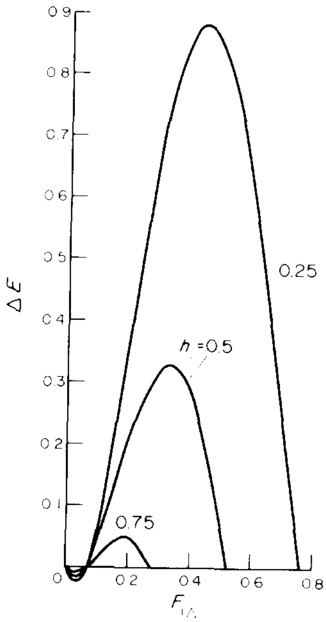

Figure 6. Energy flux for air water system.

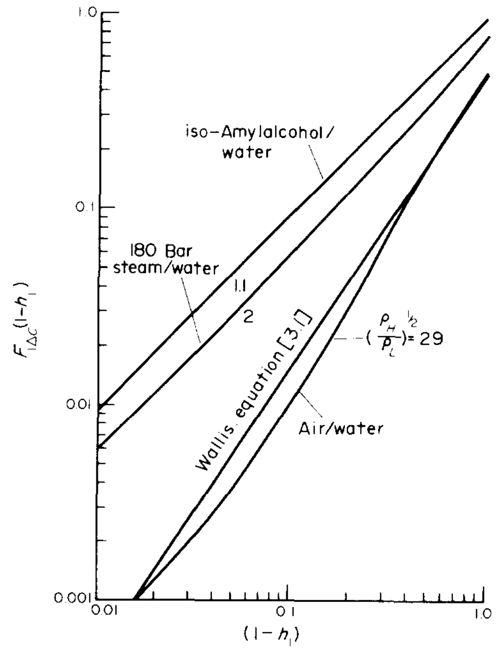

Figure 7. Prediction for onset of slugging.

$F_{1\Delta}$ is varied. It is seen that ΔE passes through a pronounced maximum and the hypothesis is that conditions such that ΔE increases as $F_{1\Delta}$, or the light phase flowrate, increases are possible but that conditions in which ΔE decreases with $F_{1\Delta}$ are improbable. Therefore the maxima of the curves indicate the critical value of $F_{1\Delta}$ where slugging begins in order to maintain the increase of ΔE with air flowrate. All this, of course, supposes that the largest possible lossless waves obtain, which may not be true when $F_{1\Delta}$ is small but which is not improbable as $F_{1\Delta c}$ is approached. The result is shown in figure 7, using the same coordinates as chosen by Wallis & Dobson, and the variation of $F_{1\Delta c}$ with the density ratio is clear. Figure 8 shows the experimental results obtained

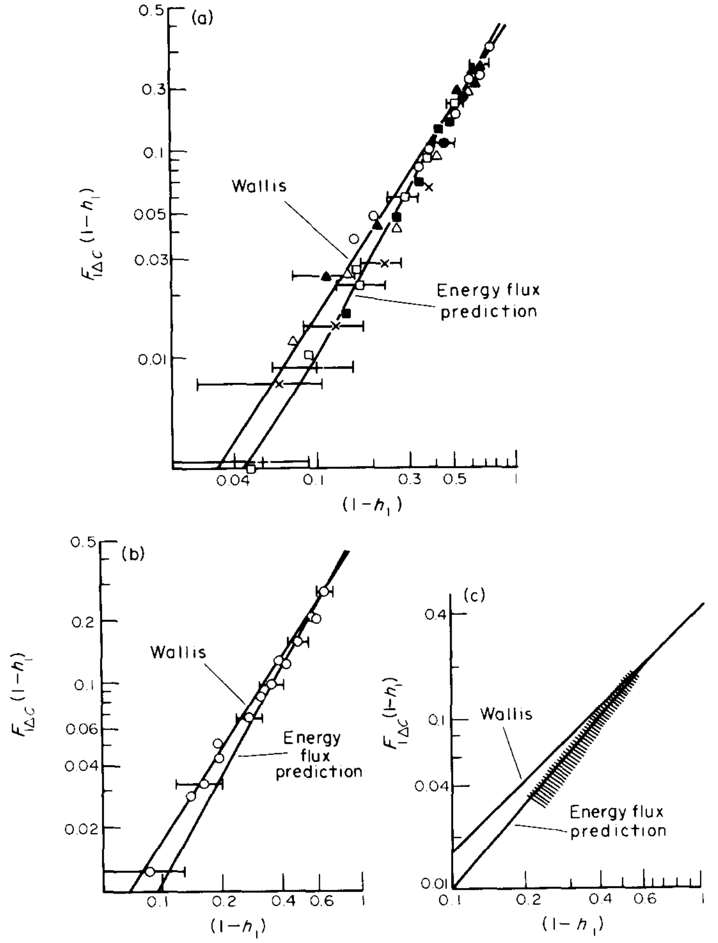

Figure 8. Comparison of energy flux prediction with experiment. (a) Data of Wallis with a paddle and wave suppression at end of channel. (b) Data of Wallis without paddle or wave suppression. (c) Data of Kordyban & Ranov.

by Wallis with various apparatus as well as results due to Kordyban & Ranov (1970). The agreement with prediction is excellent and is within the scatter of the experimental results.

4. DISCHARGE ALONG A HORIZONTAL DUCT BETWEEN CLOSED VESSELS

Figure 9 illustrates the system studied experimentally by Leach & Thompson (1975), starting with brine in one tank and water in the other. They correlated their results by

$$F_{QH} = 0.09 \qquad [4.1]$$

where F_{QH} is defined by [1.1].

The present author's assumptions concerning the interface between the two fluids is given in

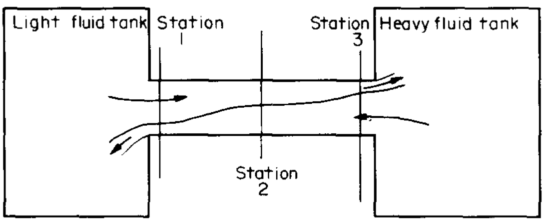

Figure 9. Definition sketch for Leach & Thompson (1975) experiment.

figure 9. Station 2 is chosen at any point in the centre region of the transfer pipe where the level may be assumed uniform. Stations 1 and 3 are at the discharge positions of the heavy and light phases, where the flow is assumed to reach the critical dynamic condition for a small disturbance. This assumption is in agreement with concepts for the flow over a broad-crested weir and was found by Gardner & Crow (1971) to apply to a similar discharge, when the light phase was assumed stationary.

The complete conditions at stations 1 and 3, especially with respect to the inflow of one of the fluids, are complex and the exact nature of the flow must remain a matter for speculation. For simplicity we will assume that the light phase is stationary at station 1, so that the condition of [2.21] can be applied. We must therefore expect to apply a correction factor or discharge coefficient to the result but the chief test of the appropriateness of the theoretical approach will be the prediction of the correct trend of F_{QH} with respect to the density ratio.

We employ the same notation as used in section 2, except that p_1 and p_2 are both taken at the level of the interface between the two fluids at station 2. We assume that

$$p_1 - p_2 = \rho_L v_{2L}^2 \qquad [4.2]$$

which can be considered as rather over-estimating the influence of the vena contracta, as the light phase enters the duct, but variations of this assumption are found to have small influence upon the result. The dynamic force balance over the area A_2' is

$$\rho_H v_{2H}^2 A_2' + \rho_H g A_2' x_2' = (p_1 - p_2)A_2' + \rho_H v_{1H}^2 A_1' + \rho_H g A_1' x_1' + \rho_L g(A_2' - A_1')y_0' + \rho_L g(h_2' - h_1')A_1'. \qquad [4.3]$$

We employ [2.6], the critical condition of [2.21] and [4.2] to achieve

$$A_2 x_2 - A_1 x_1 = \frac{\pi}{4}\frac{A_1^{\,2}}{w_1}\left[1 - \frac{A_1}{A_2}\left(1 - \frac{\rho_L}{\rho_H}\left(\frac{A_2}{1-A_2}\right)^2\right)\right] \qquad [4.4]$$

and

$$F_{QH} = \left(\frac{\pi}{4}\right)^{3/2}\frac{A_1^{\,3/2}}{w_1^{\,1/2}} \qquad [4.5]$$

where

$$x = \frac{x'}{H}, \qquad w = \frac{w'}{H}. \qquad [4.6]$$

Similar equations are written for the light phase discharge but it is noted that

$$F_{QH} = \left(\frac{\rho_L}{\rho_H}\right)^{1/2}\left(\frac{\pi}{4}\right)^{3/2}\frac{(1-A_3)^{3/2}}{w_3^{\,1/2}}. \qquad [4.7]$$

Equations [4.4], [4.5] and [4.7] have been solved simultaneously by trial and error with the result given in figure 10a. The prediction near $(\rho_L/\rho_H) = 0$ must be regarded with caution, since the discharge area for the light phase then becomes a small fraction of the total area and compressible flow effects may be important. Figure 10b compares the theoretical prediction with the data of Leach & Thompson (1975) and a line with F_{QH} equal to 0.79 times its theoretical value is drawn through the mass median of the experimental points. The discharge coefficient of 0.79 is of reasonable magnitude but it is also noted that the trend of the results with the density ratio is correctly predicted.

It is of interest to note that the theory for a horizontal duct, with a stationary upper phase and

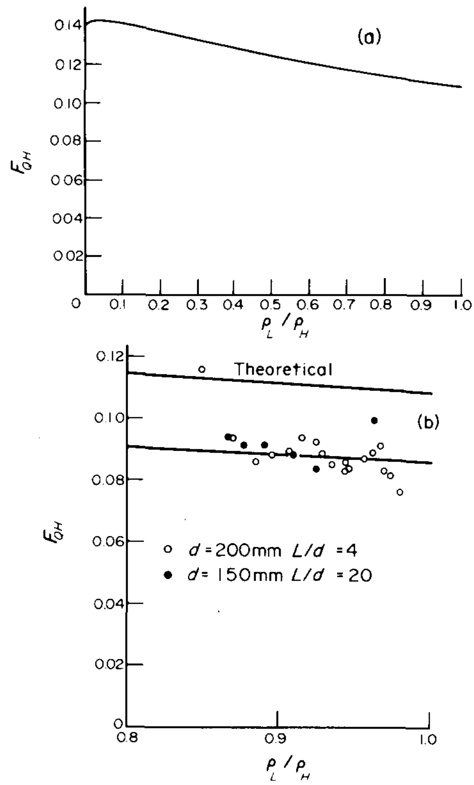

Figure 10. Prediction and experimental results for countercurrent discharge of miscible fluids along a horizontal pipe between tanks.

with the heavy phase progressively introduced and accelerated from an upstream position, where the flow is zero, to a downstream discharge position, reduces to precisely the same equations as given above for the particular case of a density ratio of unity. The theory does not then involve the same awkward assumptions with respect to the light phase. In the present example the duct runs half full at station 2 when $(\rho_L/\rho_H) = 1$ but the theory for the other problem is applicable for any depth at the upstream point as well as to any density ratio. Beij (1934) examined a roof gutter running half full at the upstream point of zero flow and found $F_{QH} = 0.093$, which compares with the extrapolation of Leach and Thompson's results to $(\rho_L/\rho_H) = 1$ of 0.086. Gardner & Crow (1971) examined the roof gutter type of problem for lower upstream levels and found closer agreement between theory and experiment than Beij.

In conclusion it can be stated that this section and the last have given additional evidence for the application of simple momentum flux equations to problems in horizontal flow but, equally important, the influence of the density ratio (ρ_L/ρ_H) on the values of modified Froude numbers, such as that of [1.1], has been demonstrated.

5. THEORY FOR THE DIFFUSION OF UNSTABLY STRATIFIED FLUIDS IN VERTICAL DUCTS

It has been postulated in the introduction that unstably stratified fluids move through each other in a fashion that can be described by the diffusion equations. Now the transfer of light fluid upwards and heavy fluid downwards implies the dissipation of energy and therefore the hypothesis is put forward that the diffusivity, D, is dependent only upon the dissipation of energy per unit mass of fluid, ϵ, and the diameter, d, of the duct. In consequence

$$D = K\epsilon^{1/3}d^{4/3} \qquad [5.1]$$

where K is constant.

This same assumption was also made recently by Baird & Rice (1975) to correlate the experimental results from seven different sources for the axial diffusion of material in a bubble column. In that case $\epsilon = \epsilon_B$ is given by

$$\epsilon_B = ug \qquad [5.2]$$

where u is the superficial gas velocity and g is the acceleration due to gravity. It was found that $K = K_B = 0.35$.

In the present case we derive an expression for the energy dissipation rate as follows. Assume that there are two distinguishable fluids with densities ρ_H and ρ_L and volumetric concentrations c_H and c_L. Assume also that volume is conserved upon mixing. If the vertical coordinate is y upwards, then we have the flux of the light fluid $-D(\partial c_L/\partial y)$ which is given potential energy $-D(\partial c_L/\partial y)\rho_L g dy$ in rising dy. Remembering that $(\partial c_H/\partial y) = -(\partial c_L/\partial y)$, we therefore find that energy dissipation per unit mass is

$$\epsilon = \frac{\Delta\rho D g}{(c_H\rho_H + c_L\rho_L)}\frac{\partial c_H}{\partial y} \simeq \frac{\Delta\rho D g}{\rho_a}\frac{\partial c_H}{\partial y} \qquad [5.3]$$

and

$$2\rho_a = \rho_H + \rho_L\,. \qquad [5.4]$$

Thus from [5.1] and [5.3]

$$D = K^{3/2}\left\{\frac{\Delta\rho g}{\rho_a}\right)^{1/2}\frac{\partial c_H}{\partial y}d^2\,. \qquad [5.5]$$

5.1 *Evaluation of* K *in* [5.5]

Mercer & Thompson (1975) carried out experiments in which a closed vessel containing brine was connected through a tube in its base with a tank of water. They measured the flux of water, Q, into the tank. Therefore

$$-D\frac{\partial c_L}{\partial y} = \frac{4Q}{\pi d^2} \qquad [5.6]$$

and

$$\frac{dc_L}{dy} = -\frac{1}{L} \qquad [5.7]$$

where L is the length of the tube.

Substituting from [5.5] and [5.7] into [5.6] we obtain

$$K = \left(\frac{4}{\pi}\right)^{2/3} F_Q^{2/3}\left(\frac{L}{d}\right) \qquad [5.8]$$

where

$$F_Q = \left[\frac{\rho_a}{\Delta\rho g d^5}\right]^{1/2} Q\,. \qquad [5.9]$$

Though it is noted that, if an average density had not been assumed, an influence of the density ratio would have been found. However, the available experimental information does not warrent an examination of the influence of the density ratio on F_Q.

Mercer & Thompson's (1975) data are plotted in figure 11 in the manner suggested by [5.8] and

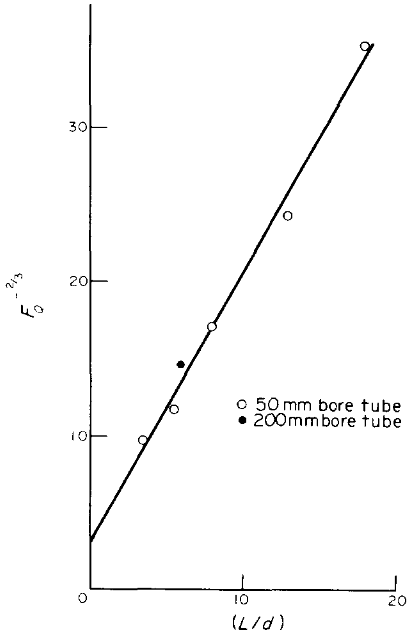

Figure 11. Ingress of light fluid up a vertical pipe into a tank of heavy phase. Results plotted according to [5.8].

it is seen that a straight line is in excellent agreement with the results, though it suggests an end effect equivalent to an L/d of about 1.7. The results substantiate that the process can be represented as one of diffusion and [5.5] gives the diffusion coefficient with $K = 0.68$ or $K^{3/2} = 0.56$.

5.2 *Predicted diffusion process in Kubie's experiment*

As described in the Introduction, Kubie conducted an ideal unsteady state experiment by suddenly connecting two unstably stratified fluids in a vertical pipe. It is instructive to predict the properties of the concentration wave in his experiment, since a simple analytical result is obtained equivalent to that for two masses containing different concentrations of diffusible matter which are suddenly brought together.

The diffusion equations is

$$\frac{\partial\left(D\dfrac{\partial c_L}{\partial y}\right)}{\partial y} = \frac{\partial c_L}{\partial t} \qquad [5.10]$$

where t is time. Substituting for D and noting that it is in terms of $(\partial c_H/\partial y)$,

$$\frac{\mathrm{d}^2 c_H}{\mathrm{d}Y^2} = 4Y\left(\frac{\mathrm{d}c_H}{\mathrm{d}Y}\right)^{1/2} \qquad [5.11]$$

where

$$Y = y\left[15K^{3/2}\left(\frac{\Delta\rho g}{\rho_a}\right)^{1/2}\mathrm{d}^2 t\right]^{-0.4}. \qquad [5.12]$$

The solution to [5.11] with the boundary conditions

$$Y = 0, \qquad c_H = 0.5\,,$$
$$Y = Y_0, c_H = 1\,, \qquad [5.13]$$
$$Y = Y_0, \qquad \frac{dc_H}{dY} = 0\,.$$

is

$$c_H - 0.5 = \left(\frac{15}{16}\right)^{0.8} Y - \frac{2}{3}\left(\frac{15}{16}\right)^{0.4} Y^3 + 0.2Y^5 \qquad [5.14]$$

which is valid in the range

$$-\left(\frac{15}{16}\right)^{0.2} \leqslant Y \leqslant \left(\frac{15}{16}\right)^{0.2}. \qquad [5.15]$$

Thus, in this case where the diffusivity varies with the concentration gradient, the extent of the concentration wave is finite.

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REFERENCES

BAIRD, M. H. I. & RICE, R. G. 1975 Axial dispersion in large unbaffled columns. *Chem. Engng J.* **9**, 171–174.

GARDNER, G. C. & CROW, I. G. 1970 The motion of large bubbles in horizontal channels. *J. Fluid Mech.* **43**, 247–255.

GARDNER, G. C. & CROW, I. G. 1971 Onset of drawdown of supernatant fluid in surge tanks. *Chem. Engng Sci.* **26**, 211–219.

KORDYBAN, E. S. & RANOV, T. 1970 Mechansim of slug formation in horizontal two-phase flow. *J. Bas. Engng* **92**, 857–964.

LEACH, S. J. & THOMPSON, H. 1975 An investigation of some aspects of flow in gas-cooled nuclear reactors following an accidental depressurization. *J. Br. Nucl. Energy Soc.* **14**, 243–250.

LONG, R. R. 1956 Solitary waves in one- and two-fluid systems. *Tellus* **8**, 460–472.

MERCER, A. & THOMPSON, H. 1975 An experimental investigation of some further aspects of the buoyancy-driven exchange flow between carbon dioxide and air following a depressurization accident in a magnox reactor. *J. Br. Nucl. Energy Soc.* **14**, 327–340.

WALLIS, G. B. & DOBSON, J. E. 1973 The onset of slugging in stratified air–water flow. *Int. J. Multiphase Flow* **1**, 173–193.

YIH, C. 1965 *Dynamics of Nonhomogeneous Fluids*. Collier-MacMillan, London.

ZUKOSKI, E. E. 1966 Influence of viscosity, surface tension and inclination angle on motion of long bubbles in closed tubes. *J. Fluid Mech.* **25**, 821–837.